ECN 3103
Industrial Organisation

5. Game Theory

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Our Plan

- **Analyze Strategic price and Quantity Competition (Noncooperative Oligopolies)**

Reference for reviewing these concepts:

Carlton, Perloff, Modern Industrial Organisation, Addison Wesley Longman, Inc, Chapter 4

Motta, Massimo, Competition Policy: Theory and Practise, Cambridge University Press, chapter 8.

GAME THEORY

- tool set for predicting outcome of interactions in which participants affect each others payoffs with their actions
- in particular, useful in small number cases
- has been applied to economics, political sciences, animal behavior, military, psychology etc
- very important tool for industrial organization, i.e. the analysis of markets with imperfect competition, and competition policy
- different solution concepts for different strategic situations as a function of timing and information:

games	*complete information*	*incomplete information*
static	Nash equilibrium	Bayes-Nash equilibrium
dynamic	Subgame Perfect equilibrium	Perfect Bayesian equilibrium

Nash Equilibrium in Static Games with Complete Information

A static game consists of

1 set of players

2 action set for each player

3 payoff function for each player which assigns number to each outcome

Example: Golden Balls

(http://www.youtube.com/watch?v=TKaYRH6E36U)

Definition: An outcome is said to be a Nash equilibrium if no player would find it profitable to deviate provided that all other players do not deviate.

how to find Nash equilibria in a game:

1 checking for every outcome whether at least one player could benefit from deviating; if not, NE found!

2 deriving best-response (or reaction) functions : Find best action of player for ALL feasible actions of rivals; NE at outcome where players actions are best responses to each other (i.e. where BR intersect)

Example: Prisoners' Dilemma

	confess	*not confess*
confess	1,1	5,0
not confess	0,5	4,4

- best response to confess is confess, best response to not confess is also confess
- unique Nash equilibrium is outcome (confess, confess)

Multiple Nash equilibria and Pareto Dominance Criterion

Definition: An Nash equilibrium Pareto dominates another equilibrium if at least one player would be better off in this equilibrium and no other player worse off.

Example: Battle of the Sexes

	opera	football
opera	3,5	0,0
football	0,2	3,3

- for both players: best response to opera is opera, best response to football is football
- Nash equilibria: (football, football), (opera, opera)
- (opera, opera) Pareto dominates

Example: Derive the Nash equilibrium of this game as a function of the parameters X > 0 and Y > 0.

	L	M	R
T	4,5	0,X	7,2
M	Y,1	2,2	9,1
B	3,10	1,6	8,8

- best response for row player to $\begin{cases} L & \text{is } T(M) \text{ if } Y < (\geq)4, \\ M & \text{is } M \\ R & \text{is } M \end{cases}$

- best response for column player to

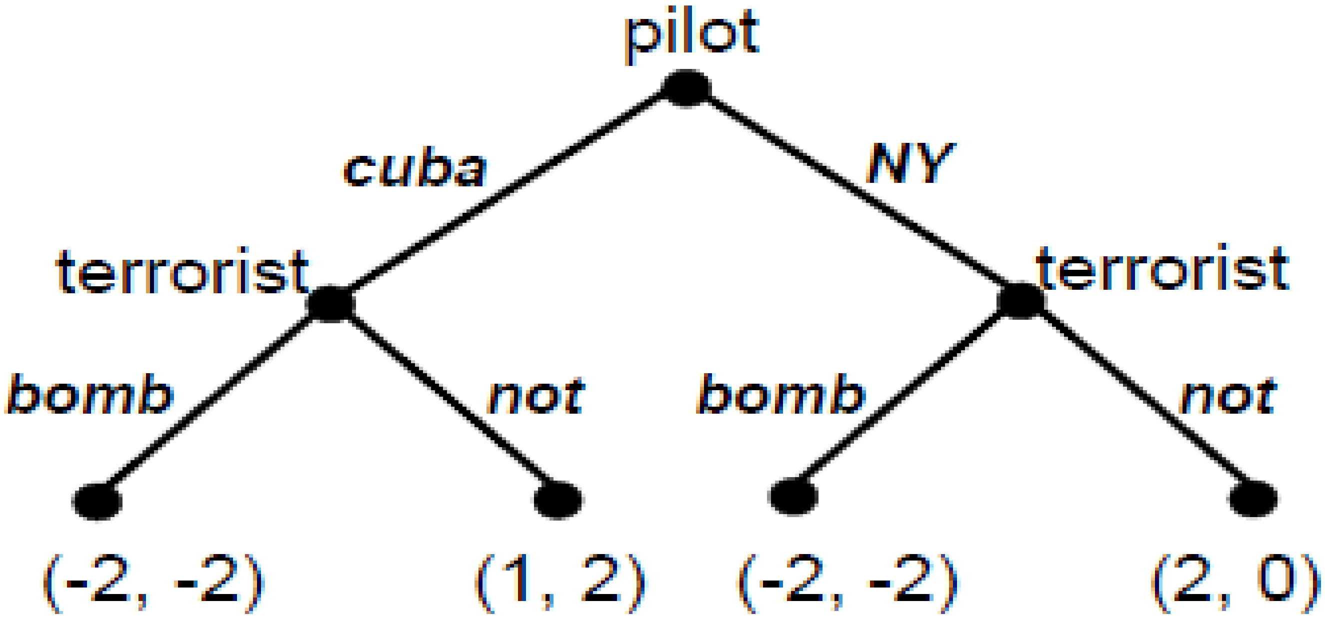
$$\begin{cases} T & \text{is } L(M) \text{ if } X < (\geq) 5, \\ M & \text{is } M \\ B & \text{is } L \end{cases}$$

- (M;M) always NE
- (T; L) NE if and only if $Y < 4$ and $X < 5$

Subgame Perfect Equilibrium in Dynamic Games with Complete Information

- A dynamic game can be represented in an extensive form specifying:
- Game tree with a starting node, decision nodes, terminal nodes, branches linking each decision nodes
- List of players
- For each decision node: name of player entitled to choose action and complete set of actions
- Payoffs at each terminal node

Example: Pilot and Terrorist



- note this game is sequential, the terrorist can observe what the pilot is doing before he acts
- simultaneous moves mean players cannot observe rivals move

Definition: A strategy is a complete plan (list) of action for each decision node that the player is entitled to choose an action.

strategy=contingent plan of action that you could leave at your lawyer and your lawyer could play for you

Definition: A subgame is a decision node from the original game at which a player is called upon to act, along with the decision nodes and terminal nodes following this node.

Definition: An outcome is said to be a subgame perfect equilibrium (SPE) if it induces a Nash equilibrium in every subgame of the original game.

Finding SPE by backward induction: Look for strategies that are NE in each subgame starting from the last subgame of the game

Repeated Games

- repeated game=one-shot game that is identically repeated several times

- players observe outcomes of past rounds which creates a public history of the game

- history at point t is the list of all outcomes in periods 1..t- 1

- strategy in a repeated game is list of actions that player takes in each period t = 1; 2; ::T where each action is based on period t history

- strategy assigns an action to each possible history in period t

- for each possible history in period t a new subgame starts

- two classes of repeated games: finite horizon and infinite horizon

- finite horizon: players know end date

- infinite horizon: players do not know end date

Example: Prisoners' Dilemma X2

suppose this game is played twice:

	confess	not confess
confess	1,1	5,0
not confess	0,5	4,4

What is the Subgame Perfect equilibrium?

- backward induction: NE in second period (confess, confess) independent of outcome in first period
- given NE in second period, each player maximises short-run payoff in first round
- outcome in both periods is (confess, confess)

General result for game with finite horizon:

If the stage game has a unique Nash equilibrium, then for any finite number of repetition, the repeated game has a unique subgame perfect equilibrium: NE strategies are played in every stage.

- if the base game has a unique NE, repetition cannot change the outcome of a game
- players anticipate that outcome of current round has no impact on future rounds
- players maximise payoff from current rounds only
- strategies are not intertemporally linked

Repeated Games with Infinite Horizon

- infinite horizon means end of game is not deterministic
- suppose δ is probability that game continues after each round
- backward induction can no longer be used to solve for SPE
- however, subgame perfectness concept still works

How many subgame are there with an infinite horizon?

- there are as many subgames beginning at t as there are possible histories at that date
- strategies can be very complex
- number of strategically different subgames can be finite, though, which allows to devise simple strategies and check if they are SPE
- we consider repeated games with infinite horizon in detail in -Collusion and Cartel Policy

Oligopoly Theory

analyses markets with a small number of firms and strategic interaction

cases between monopoly and perfect competition

reasons for small number of firms in markets: market size, increasing returns to scale, sunk cost of entry

we will look at the main models of oligopoly theory, their applications, differences and how to apply them to real world industries

1 Cournot model of quantity competition

2 Bertrand model of price competition

3. Stackleburge

Cournot model of quantity competition

Antoine-Augustin Cournot (1801-1877)

- doctorate in mechanics and astronomy, founding father of mathematical economics
- his main work in Economics (1838) formalized the monopolists problem
- extended the analysis to multiple firms (oligopoly) under the assumption that a firm believes that the rivals dont change their quantity
- his solution is identical to the Nash equilibrium solution of the oligopoly problem

Cournot's Model of quantity competition

- market with two firms i = 1; 2 with constant marginal cost c_i
- inverse market demand for a homogeneous good: P(Q)
- where Q is the sum of both firms production levels, Q = q1 + q2
- firms choose their quantity simultaneously (static game)
- firms maximize their profits

$$\Pi_1(q_1, q_2) = (P(q_1 + q_2) - c_1)q_1$$
$$\Pi_2(q_1, q_2) = (P(q_1 + q_2) - c_2)q_2$$

- optimal quantity choice of firm depends on quantity choice of rival; if rival does not produce, firm selects monopoly quantity

Definition: A Nash Equilibrium of the Cournot model is a (q_1^*, q_2^*)

for a given q_2^*, q_1^* solves

$$max_{q_1} \Pi_1(q_1, q_2^*) = (P(q_1 + q_2^*) - c_1) q_1$$

for a given q_1^*, q_2^* solves

$$max_{q_2} \Pi_2(q_1^*, q_2) = (P(q_1^* + q_2) - c_2) q_2$$

- given the other firm's optimal quantity each firm maximises its profit over the residual inverse demand
- in equilibrium no firm can increase profits by changing its output level

- two first order conditions that implicitly determine (q_1^*, q_2^*)

$$\frac{\partial \Pi_1}{\partial q_1} = \frac{\partial P(Q)}{\partial Q} q_1 + P(Q) - c_1 = 0$$

$$\frac{\partial \Pi_2}{\partial q_2} = \frac{\partial P(Q)}{\partial Q} q_2 + P(Q) - c_2 = 0$$

- marginal and infra-marginal consumer effect
- with linear demand and constant MC: unique solution to equation system
- negative externality between firms (see Q in first/second term)
- each equation describes one firm's optimal behaviour given any rival's quantity; in equilibrium the quantities are mutually consistent
- first-order conditions are best response or \reaction" functions

- we can rewrite the first-order condition of a firm in terms of price-cost margins:

$$\frac{P(Q) - c_i}{P(Q)} = -\frac{\partial P(Q)}{\partial Q} \frac{q_i}{P(Q)} \frac{Q}{Q} = \frac{s_i}{\epsilon_{Q,p}}$$

Where $s_i = q_i/Q$ is firm i 's market share

- a firm's equilibrium market share decreases in its marginal cost

- the more inelastic market demand, the higher the equilibrium market price

Example: $P(Q) = 100 - Q$, $c_1 = c_2 = c = 10$

- profit function of firm 1:

$$\Pi_1(q_1, q_2) = \underbrace{(100 - q_2 - q_1)}_{\text{residual inverse demand}} q_1 - 10 q_1$$

- like in monopoly solution: optimal quantity of firm sets

MR of residual inverse demand=MC

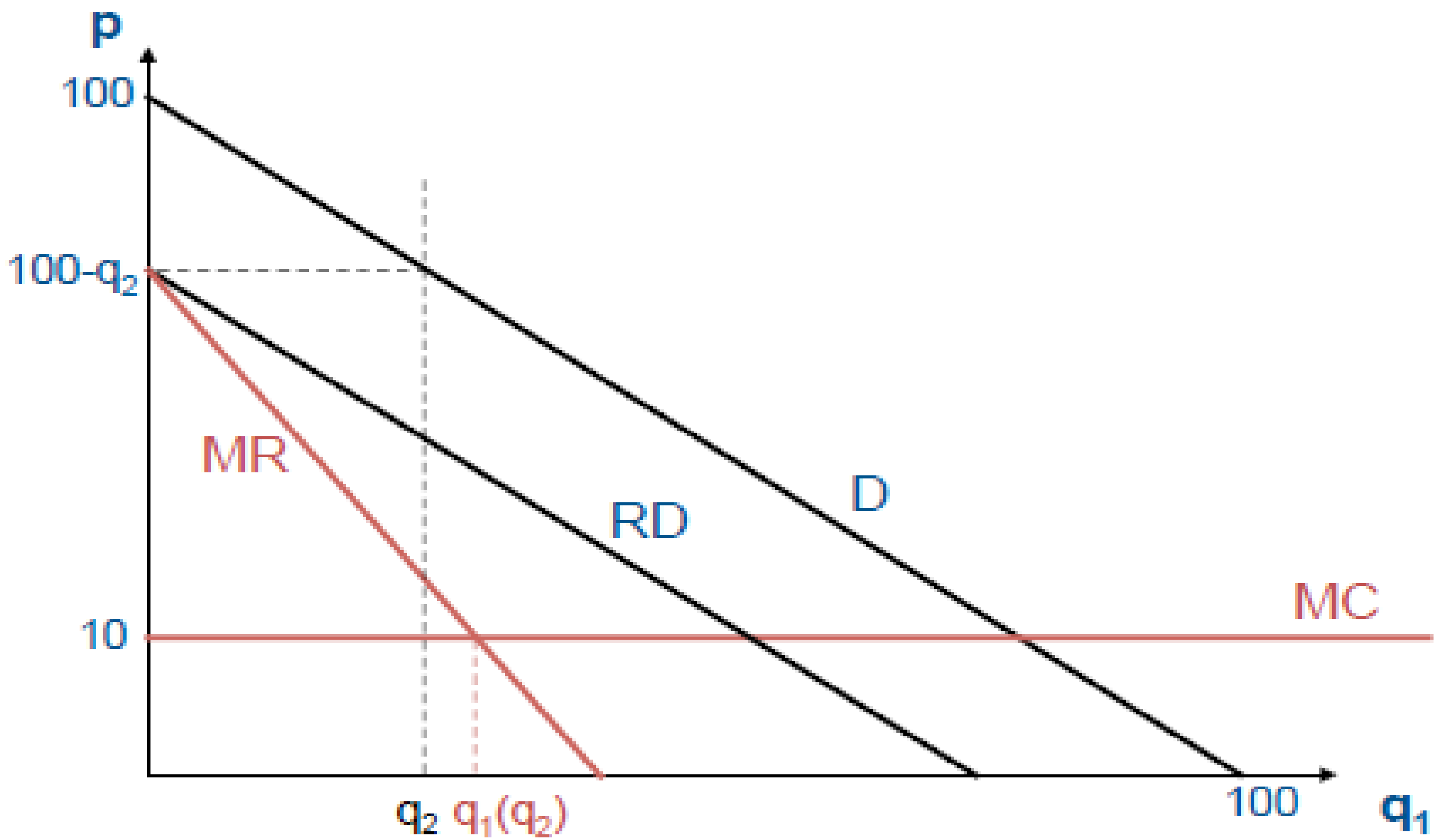
that is,

$$MR(q_1) = 100 - q_2 - 2q_1 = 10$$

which yields

$$q_1 = \frac{90 - q_2}{2} \quad \text{and} \quad R_1(q_2) = max\{\frac{90 - q_2}{2}, 0\}$$

- best response $R_1(q_2)$ gives optimal quantity of firm 1 for any quantity of firm 2
- graphical derivation of best response to q_2:



- repeat this for any q_2 to construct reaction curve R1(q_2) for firm 1

- in our example: reaction function is linear and decreasing in q_2 with

$$q_1(q_2 = 0) = q^m = 45 \text{ and } q_1(q_2 \geq q^c = 90) = 0$$

- with these two points, we can draw the best response functions for both firms in a q_2 diagram

- Nash equilibrium at intersection of these functions (see next slide)

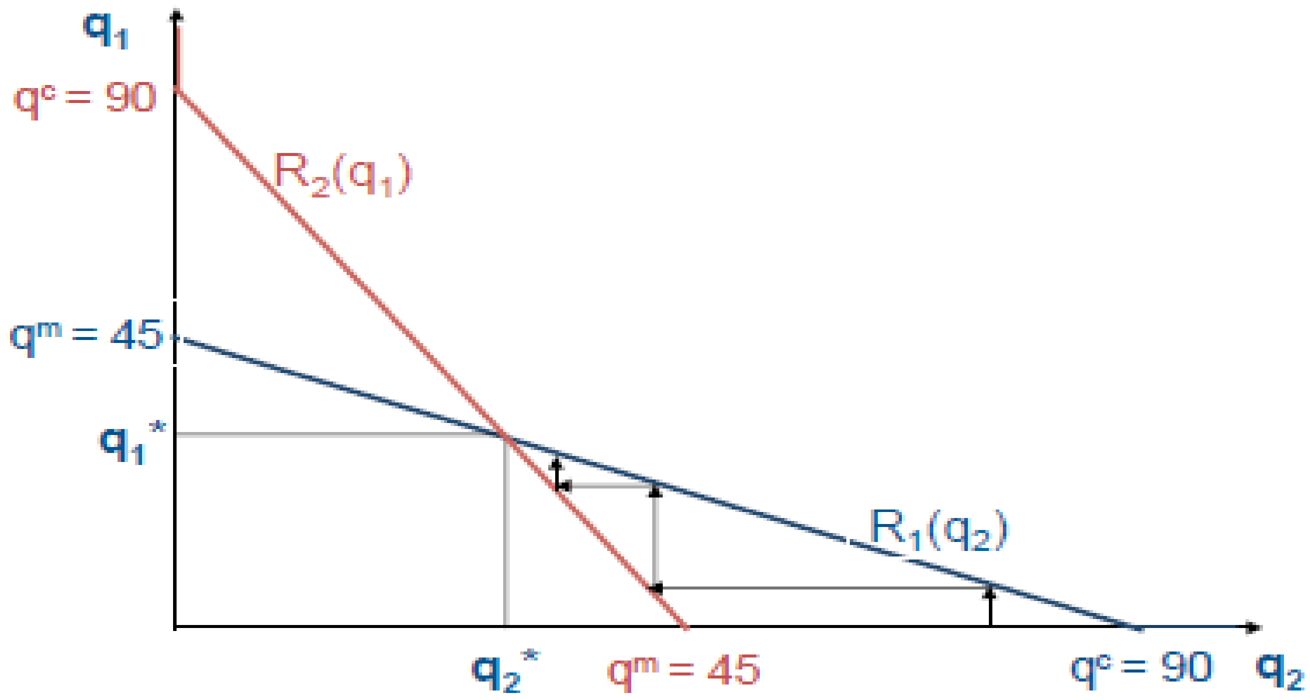
- reaction function of firms 2 is given by

$$R_2(q_1) = max\{\frac{90 - q_1}{2}, 0\}$$

to cross reaction functions, substitute one reaction function in other one and solve for quantity:

$$q_1 = \frac{90 - \frac{90 - q_1}{2}}{2} \quad \text{or} \quad q_1^* = 30$$

Graph: Best response functions and Cournot Nash equilibrium



- dynamic interpretation possible
- stability guaranteed with linear demand and constant MC

Since $R_2(q_1^* = 30) = \frac{90-30}{2} = 30$ we get

$$Q^* = 60 \quad \text{and} \quad P^* = 40$$

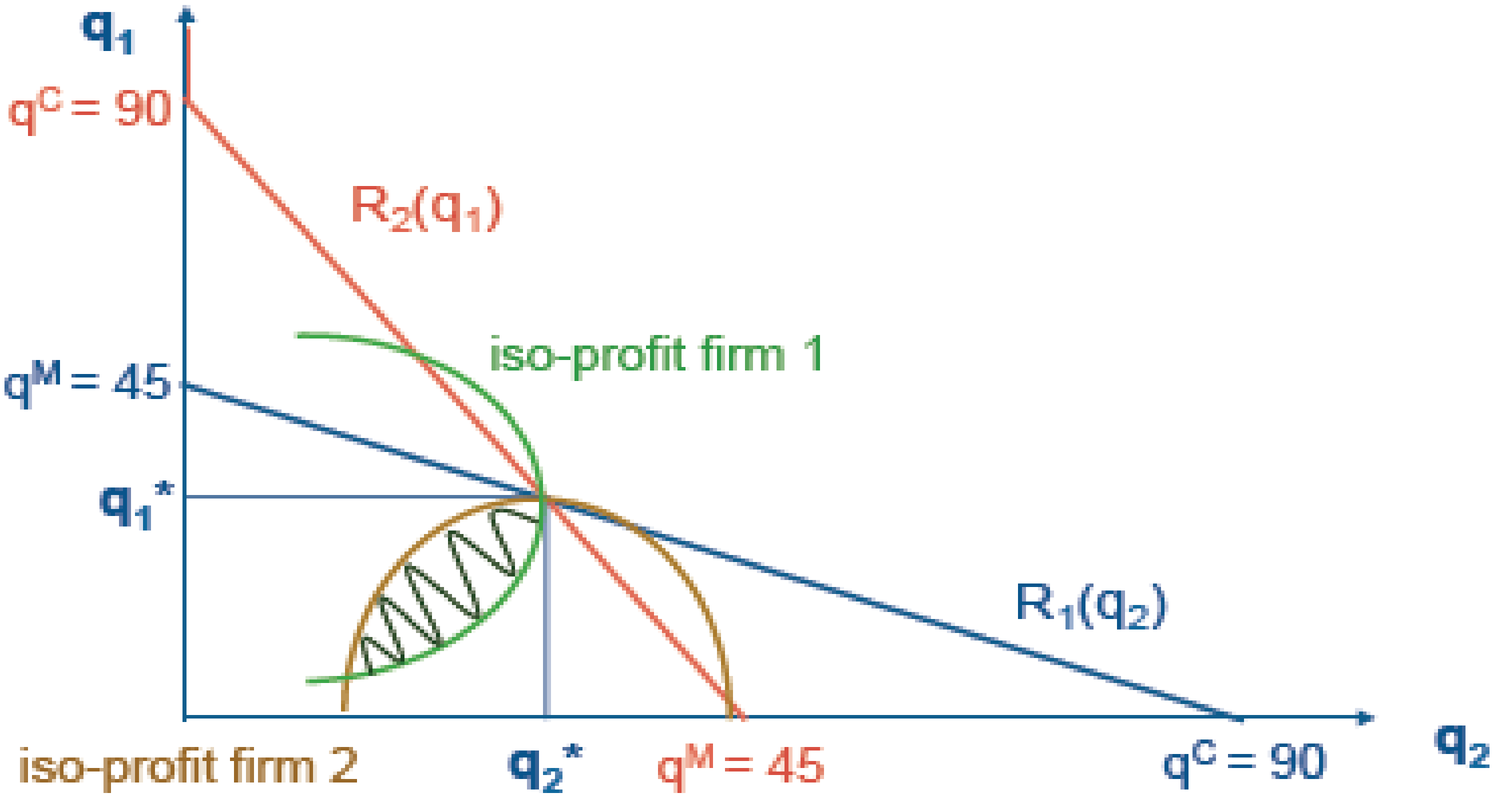
- in other words, Cournot duopoly yields market allocation strictly between perfect competition (Qc = 90; Pc = 10) and monopoly (Qm = 45; Pm = 55)

Firms' profits in Nash equilibrium
iso-profit curve=all combinations (q1; q2) yielding the same profit πi
for example for firm 2: $\pi_2 = (90 - q_1 - q_2)q_2$ or

$$q_1 = 90 - q_2 - \frac{\Pi_2}{q_2}$$

draw them in diagram with reaction functions through NE quantities

Graph: Profits in Cournot Nash equilibrium



- firms could Pareto-improve on NE if they both produced less (shaded area)
- however, each firm would have unilateral incentive to deviate to its BR

- individual firm does not take into account the negative effect of its own quantity increase on market price and rival profits

- in other words, firms exert negative externality on each other by individually producing too much

- this externality leads to the result that industry production in an oligopoly is higher compared to a monopoly

Example: N-firm Cournot Oligopoly $P(Q) = 100 - Q$, n firms, $c_i = c$

- suppose firm i produces q_i and remaining firms Q_{-i} such that $Q = q_i + Q_{-i}$

- profit function of firm i :

$$\Pi_i\ (q_i;\ Q_{-i}) = (100 - q_i - Q_{-i})\ q_i - cq_i$$

price affected by Q_{-i} not distribution of production among those firms

first-order condition for firm i :

$$\frac{\partial \Pi_i}{\partial q_i} = 100 - Q_{-i} - 2q_i - c = 0$$

$$q_i = \frac{100 - c - Q_{-i}}{2}$$

- reaction function of firm i is $R_i(Q_{-i}) = Max\{\frac{100 - c - Q_{-i}}{2}, 0\}$

- as firms are identical, we can focus on symmetric NE quantities

q_1*= q_2*= ..q_n*= q*

at the intersection of the best response functions it has to hold:

$$q^* = \frac{100 - c - (n-1)q^*}{2}$$

$$100 - c - (n+1)q^* = 0$$

-the NE quantity is thus

$$q^* = \frac{100 - c}{n + 1}$$

- the resulting total output and market price are:

$$Q^* = \frac{n(100 - c)}{n+1} \quad \text{and} \quad p^* = 100 - Q^* = c + \frac{100 - c}{n+1}$$

- hence, individual production decreases, total industry output increases and the price decreases as the number of firms goes up
- equilibrium price approaches perfect competition level as n becomes large
- Cournot oligopoly outcome transitions continuously from monopoly allocation to perfect competition as number of firms in industry increases

Bertrand model of price competition

Joseph Bertrand (1822-1900)

- French mathematician, worked on thermodynamics and probability theory
- read Cournot's work but mistook quantities for prices
- came up with alternative solution to oligopoly theorem: the Bertrand paradox

Bertrand's Model of price competition

- consider price competition among two firms ($i = 1; 2$) selling homogeneous good
- downward sloping market demand $D(p)$, with $D(p) < 0$
- constant, symmetric marginal cost $c1 = c2 = c$
- static game: firms set prices simultaneously

- rationing rule of demand:

1 lowest priced firm wins all demand at its price

2 if prices are tied, each firm gets half of market demand at this price

- firm i's individual demand is

$$D_i(p_i, p_j) = \begin{cases} D(p_i) & \text{if } p_i < p_j, \\ D(p_i)/2 & \text{if } p_i = p_j \\ 0 & \text{otherwise.} \end{cases}$$

- - firm i's profits

$$\Pi_i(p_i - c)D_i(p_i, p_j)$$

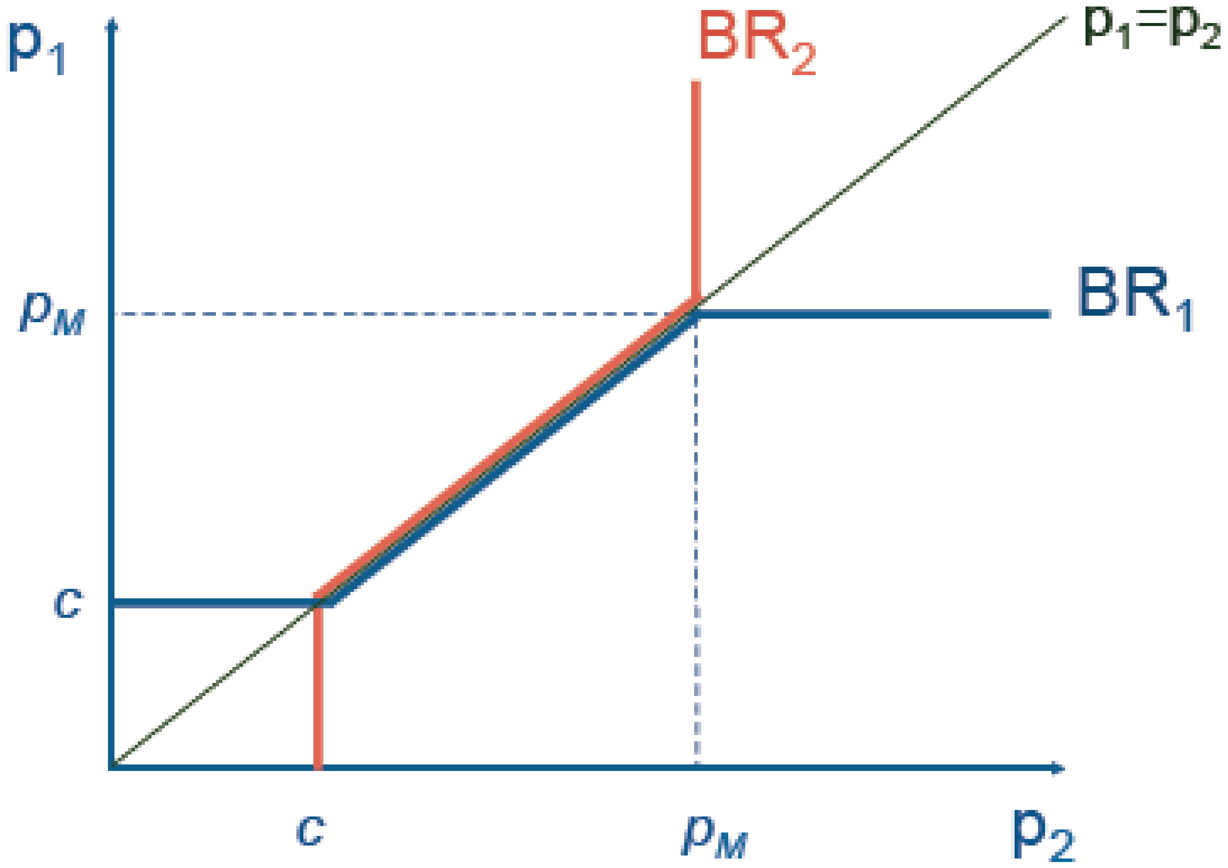
- What is Nash equilibrium (NE) of this game?

- we need to derive best responses
- let pm be the monopoly price, pm = argmaxp(p- c)D(p)
- firm i 's best response is

$$R_i(p_j) = \begin{cases} p^m & \text{if } p_j > p^m, \\ p_j - \epsilon & \text{if } c < p_j \leq p^m \\ c & \text{if } p_j \leq c. \end{cases}$$

- for rival prices above cost, each firm has incentive to undercut rival to get the whole demand
- if rival prices below cost, firms makes losses when it attracts demand; firm better off charging at cost level

Graph: Nash equilibrium in prices



- reaction functions are upward-sloping
- Nash equilibrium is p1 = p2 = c

Bertrand Paradox: Under static price competition with homogenous products and constant, symmetric marginal cost, firms price at the level of marginal cost and make no economic profits.

- two competitors in a market are sufficient to guarantee perfectly competitive outcome

-price and profit is not function of number of competitors

- benchmark result depends on four assumptions:

1 identical firms with same cost structure

2 constant marginal cost, no capacity constraints

3 static game, one-off competition

4 homogenous product, no product differentiation

Product differentiation: Imperfect Substitutes

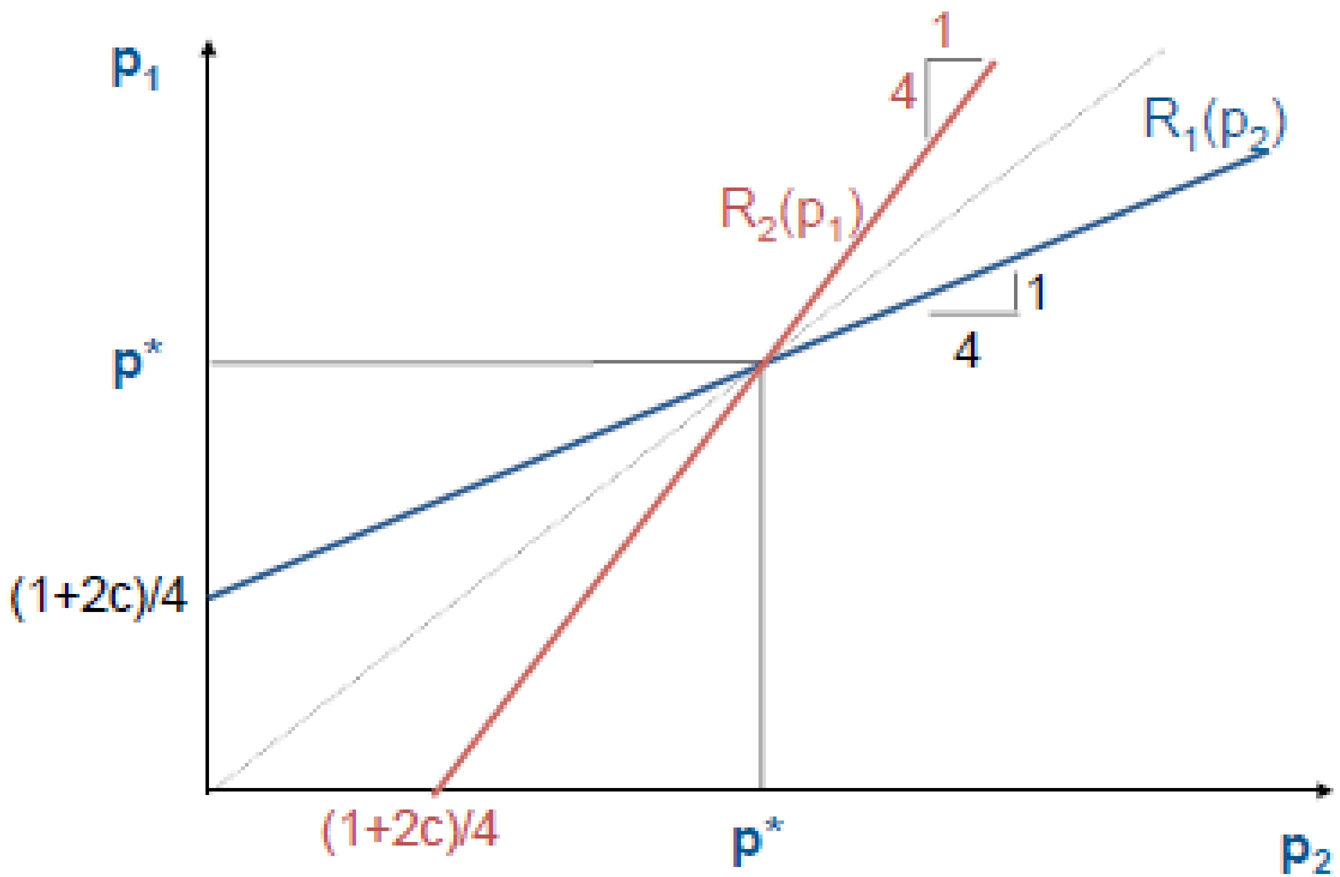
- in the case of substitutes: positive cross-price demand elasticity
- when rival's product becomes more expensive, firm's demand increases
- simple linear demand system for substitute products

$$\max_{x_1, x_2} \quad U = x_1 + x_2 - \frac{1}{2(1+\gamma)}\left(x_1^2 + 2\gamma x_1 x_2 + x_2^2\right) - p_1 x_1 - p_2 x_2$$

where $\gamma \in [0; 1]$ is the degree of substitutability ($\gamma = 0$: independent products $\gamma = 1$: perfect substitutes)

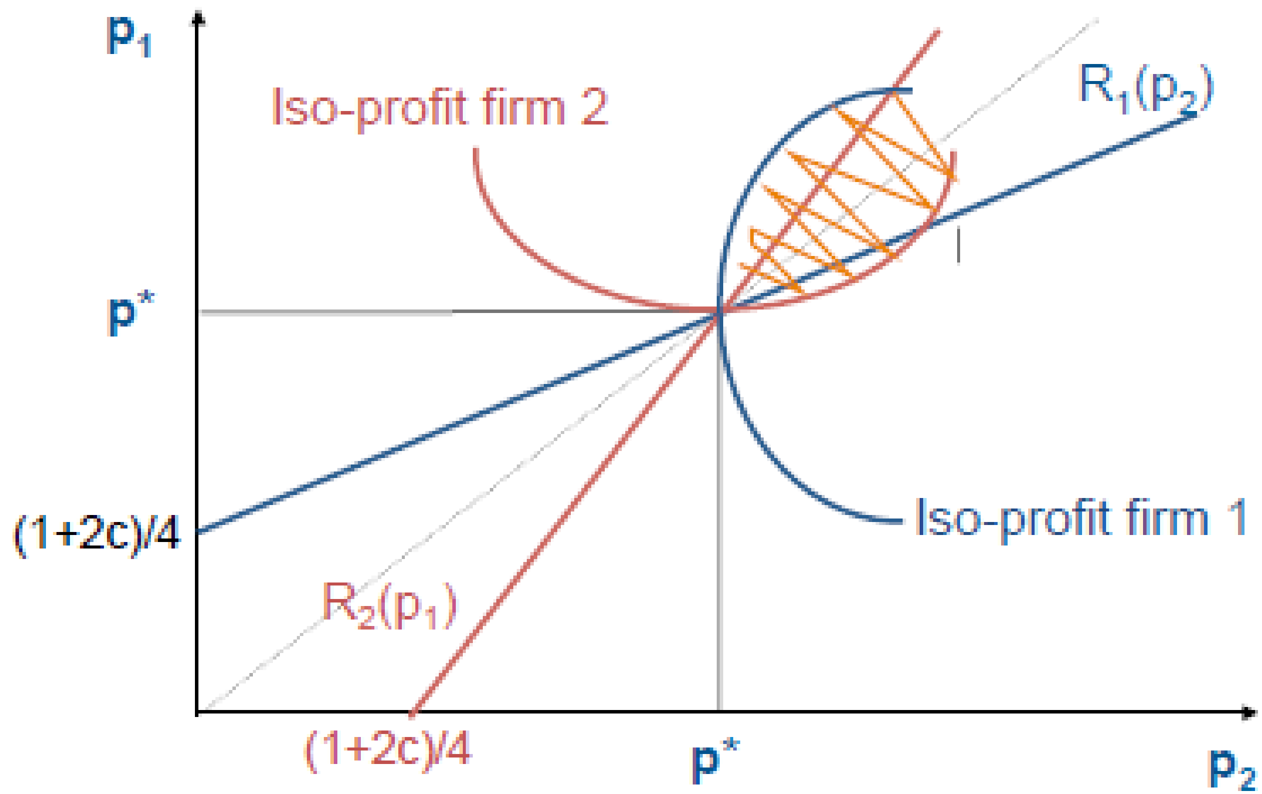
- yields demand function for firm i given its price and its rival j 's price

Graph: Price competition with imperfect substitutes



- note that undercutting rival's price only occurs if rival would charge price above equilibrium level

Graph: Profits in Nash equilibrium in prices



- from joint profit point of view, each firm is charging a too low price
- firms could be better off if both increased prices but none has unilateral incentive (like in the Prisoners' dilemma)

Product Differentiation: Complementary Products

- in the case of complements: negative cross-price demand elasticity

- demand for one firm decreases as the other firm raises its price

- with strict complements: consumer only gets value if he/she consumes both products at the same time

- demand depends on system price, the sum of prices of all system goods $D = D(p1 + p2)$

The Stackleberg Leader-Folower Model

Heinrich Von Stackelberg (1952) presented the third important Oligopoly model in 1934. In the Stackelberg model, firms set output, and one firm acts before the other.

The leader first pick its output level and then the other firms are free to choose their optimal quantities given their knowledge of the leader's output.

In some industries, historical, institutional, legal or costing factors determines which firm is the first mover.

For example, the firm that discovers a new product has a natural first-mover advantage.

Use the example from Cournot with the cost of the first 50% lower.